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Twisted supersymmetry and the topology of theory space

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Abstract: We present examples of four-dimensional, non-supersymmetric field theories in which ultraviolet supersymmetry breaking effects, such as bose-fermi splittings and the vacuum energy, are suppressed by $(\alpha/4\pi)^N$, where α is a weak coupling factor and N can be made arbitrarily large. The particle content and interactions of these models are conveniently represented by a graph with sites and links, describing the gauge theory space structure. While the theories are supersymmetric "locally" in theory space, supersymmetry can be explicitly broken by topological obstructions.

Keywords: Field Theories in Higher Dimensions, Supersymmetry Breaking, Supersymmetric Effective Theories.

Contents

1. Introduction

We do not know how Nature chooses the pattern of gauge symmetries and matter fields that describes the relativistic quantum mechanics of our low energy world. For a variety of reasons [1]–[4], it is interesting to consider a class of four-dimensional gauge theories described by graphs of "sites" and "links". A graph consists of a number of points, or "sites", (i), for $i = 1, \ldots, N_{\rm tot}$. Some pairs of sites are connected by one or more lines, which are called links. The links between the pair of connected sites (i) and (j) are denoted by $(i, j)_\alpha$, for $\alpha = 1, \ldots, n_{ij}$.

We associate a field theory with a graph by assigning gauge symmetries and matter fields as follows. A gauge group G_i is associated with each site (i); the full gauge symmetry G of the theory is then $G = \prod_i G_i$. We may think of the gauge fields for G_i as residing on the site (i) in the graph. In addition there may be additional fields on the site (i) that transform non-trivially under G_i and trivially under the rest of the gauge symmetry. Finally, with each link $(i, j)_\alpha$ in the graph, we associate a matter field transforming non-trivially under irreducible representations of $G_i \times G_j$ and trivially under the rest of the gauge group.

All gauge theories can be described in this way with only a single site associated with the full gauge group. But this picture becomes nontrivial and interesting when there are several sites. In this case, the condition that all matter fields are either site fields or link fields is a non-trivial constraint on the matter content of the theory. "Moose" or "Quiver" models provide simple examples of this class of theories in which the G_i are unitary groups, and the link fields transform as bifundamentals under the linked gauge subgroups.

We will refer to the graph associated with a given field theory as the "theory space" T of the gauge theory. In previous work [5], we presented examples of theories where theory space can be transmuted dynamically into field-theoretic spatial dimensions. Related ideas were presented in [6, 7]. This happens when the link fields higgs the gauge groups they

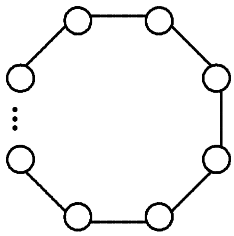

Figure 1: Theory space for a $U(1)^N$ model.

touch. The effective action in this phase contains "hopping" interactions, allowing field excitations to move from site to site. The theory space then becomes a picture of discretized extra dimensions. Among other things, this "deconstruction" of extra dimensions provides an ultraviolet completion of higher-dimensional gauge theories [5]: at energies far above the higgsing scale the extra dimensions can melt away into asymptotically free, perturbative four-dimensional dynamics.

More interestingly [5], these constructions show that the dynamics of four-dimensional theories characterized by theory spaces can be surprisingly rich, with qualitatively new possibilities beyond those of field-theoretic extra dimensions. For example, in [8], a new class of realistic models of electroweak symmetry breaking were constructed, with a naturally light Higgs field and perturbative new physics at the TeV scale, but without supersymmetry. More recently in [9], the successful unification of gauge couplings in the supersymmetric standard model was accelerated to occur at energies much lower than the usual grand unification scale. Other applications have been pursued in [10]–[19].

The sense in which theory space is a generalization of "geometric" space is perhaps best appreciated in the context of supersymmetric models [5, 10], which typically have moduli spaces of vacua. The emergence of extra dimensions from theory space happens only in certain regions of the moduli space, while in other regions no geometric interpretation is possible. As an example, consider an $\mathcal{N}=1$ $U(1)^N$ supersymmetric gauge theory whose theory space is shown in figure 1. The link Φ_i between the ith and $(i+1)$th site represents a chiral superfield with charge $(+1,-1)$ under $U(1)_i \times U(1)_{i+1}$. This theory has a classical moduli space of vacua with $\Phi_1 = \cdots = \Phi_N = \phi$, characterized by the gauge invariant operator $(\Phi_1 \cdots \Phi_N)$. For $\phi \neq 0$, the gauge symmetry is higgsed to the diagonal U(1), with all but one of the gauge multiplets becoming massive. For simplicity we will take all gauge couplings equal $g_i = g$. At energies $E \ll g\phi$ the physics of the model is the same as that of a five-dimensional supersymmetric U(1) gauge theory compactified on a circle of radius $R \equiv N/(2\pi g|\phi|)$. Here the fifth dimension is discrete, with a lattice spacing $a \equiv 1/(g|\phi|)$. The mass matrix for the vector multiplets is

$$M_{ij}^2 = \frac{1}{a^2}\left(2\delta_{i,j} - \delta_{i,j+1} - \delta_{i,j-1}\right), \tag{1.1}$$

where we identify i with $i+N$. The eigenvalues are

$$m_n^2 = \left(\frac{2}{a}\right)^2 \sin^2\left(\frac{na}{2R}\right), \qquad -\frac{N}{2} < n \leq \frac{N}{2} \tag{1.2}$$

which reproduces the Kaluza-Klein spectrum of a fifth dimension for the modes much lighter than $1/a$. Since ϕ is a modulus, so is the radius R of the generated dimension. Additional superpotential interactions would stabilize this modulus.

While the physics is that of an extra dimension for $E \ll g\phi$, for $E \gg g\phi$ this interpretation breaks down and the physics of the unbroken $\mathrm{U}(1)^N$ gauge theory is recovered. In particular, at the origin of the classical moduli space $\phi = 0$, no extra dimension is generated at any energy.

This $\mathrm{U}(1)^N$ model becomes ill-defined at high energies where the U(1) gauge couplings are near their Landau poles, but asymptotically free theories may be constructed as well. For example replace the U(1) gauge groups by $\mathrm{SU}(k)$ and the Φ_i by bi-fundamental fields. In this case there is a much larger moduli space of D-flat directions, including the directions where $\Phi_i = \phi_i \mathbf{1}_k$, characterized by the gauge-invariant operators $\det \Phi_i$. In regions of the classical moduli space where the $\phi_i \neq 0$, geometric space is generated as before. But there are many regions in classical moduli space with no extra-dimensional physics, for example along the direction where only $\phi_1 \neq 0$. As more of the ϕ_i are turned on, the full circular geometric space is built up. If only $\phi_1, \ldots, \phi_{N/2} \neq 0$, half of the theory space, corresponding to the higgsed links, turns into a finite interval fifth dimension, while the other half does not.

These examples show that, at least in this class of models, geometric extra dimensions are not fundamental, but merely an appropriate low energy description of the physics of the four-dimensional gauge theory on some parts of the classical moduli space. What is more fundamental is the underlying gauge and matter content of the four-dimensional theory, encoded in the theory space itself.

The structure of field theories characterized by theory spaces is largely unexplored. In this paper, we show that theory space can have interesting topological properties whether or not geometric space arises. This allows us to construct non-supersymmetric field theories where ultraviolet supersymmetry breaking effects can be suppressed by arbitrarily many perturbative loop factors, allowing for the generation of large hierarchies of scales in a novel way. On the portions of the classical moduli space (if any) where geometric extra dimensions are generated, this way of breaking supersymmetry is equivalent to the Scherk-Schwartz mechanism [20]. However, some of the most interesting aspects of this way of breaking supersymmetry occur at the origin of the classical moduli space where no extra-dimensional Scherk-Schwartz interpretation is possible.

2. Twisted supersymmetry

Let us begin with a simple example by returning to our $\mathrm{U}(1)^N$ model. In addition to the gauge interactions, the bare lagrangian has the gauge-Yukawa couplings

$$\sum_{i=1}^{N} \sqrt{2}\phi_i^* \left(h_i \lambda_i - h'_{i-1}\lambda_{i-1}\right)\psi_i + \text{h.c.}\,, \tag{2.1}$$

where ϕ_i, ψ_i are the scalar and fermionic components of Φ_i, λ_i are the gauginos, and we identify the index 0 with N. Supersymmetry requires that these Yukawa couplings are equal to the gauge couplings: $h_i, h'_i = g_i$. However, we can always redefine the fields $\phi_i, \psi_i, \lambda_i$ by

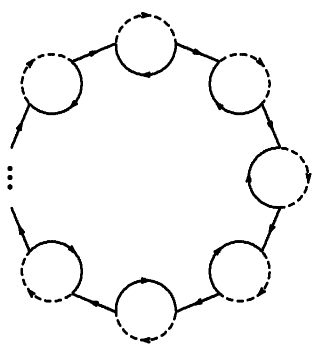

Figure 2: The leading diagram contributing to the vacuum energy.

rephasing them, so that while the magnitudes of $|h_i|, |h'_i|$ are equal to the gauge couplings g_i, their phases are non-zero. Let us the consider the more general possibility that the couplings h_i, h'_i are equal in magnitude to the g_i, but with arbitrary phases. By rephasing the fields we can remove such phases *locally* in theory space; for example we can rephase fields so that $h_1 = g_1, h'_0 = g_0$. This rephasing will in general modify the coefficient of a θ-term in the lagrangian, but since such a term is supersymmetric, it won't concern us here. However there is one phase that is physical and cannot be removed by field redefinitions. Just as the Jarlskog parameter provides a rephase-invariant measure of CP violation, we can characterize supersymmetry breaking by the rephase invariant quantity J:

$$J \equiv \prod_{i=1}^{N} h_i^* h'_{i-1} \equiv g_1^2 \cdots g_N^2 V \,, \tag{2.2}$$

where $V = e^{2i\theta}$ is a phase. If θ is non-zero, supersymmetry is explicitly broken. But this breaking is non-local, involving *all* of the Yukawa interactions. Therefore, any supersymmetry breaking effect must involve J and will be suppressed at large-N. For example if we expand around the vacuum in which $\phi = 0$, the first non-zero contribution to the vacuum energy comes from the $N + 1$ loop diagram of figure 2 plus its superpartners. Similarly the leading contribution to scalar masses arises at N loops. Cancellation between bosons and fermions, which would normally ensure the absence of any dependence on large momenta, is imperfect because of the global twist which breaks supersymmetry, proportional to $(\mathrm{Re}\, J - g_1^2 \cdots g_N^2)$. Consequently the vacuum energy and scalar masses depend sensitively on the details of the UV physics, which we represent here by a cut-off M. The resulting vacuum energy and scalar masses are then given by

$$\begin{aligned} \Lambda &\sim \sin^2\theta \left(\frac{g_1}{4\pi}\right)^2 \cdots \left(\frac{g_N}{4\pi}\right)^2 \frac{M^4}{16\pi^2} \,, \\ m_\phi^2 &\sim \sin^2\theta \left(\frac{g_1}{4\pi}\right)^2 \cdots \left(\frac{g_N}{4\pi}\right)^2 M^2. \end{aligned} \tag{2.3}$$

Since these are UV effects the couplings are to be evaluated at the scale M. The sign and precise values of these quantities depend on the details of the UV physics; here we have indicated the natural size assuming the cut-off procedure preserves local supersymmetry in theory space.

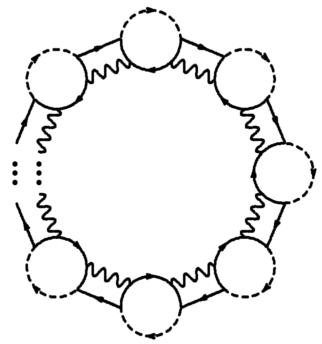

Figure 3: The same as figure 2 for a non-Abelian theory.

The breaking of supersymmetry through arg J is clearly explicit: there are no conserved supercharges and there is no goldstino. Nevertheless as long as supersymmetry is preserved locally in theory space, broken only by a global "twist" in the couplings, supersymmetry breaking is controllably small, parametrically exponentially small in N.

The same phenomena occurs for the $SU(k)^N$ model. In this case, the leading supersymmetry breaking diagrams are shown in figure 3, and the vacuum energy and scalar masses are given by

$$\Lambda \sim \sin^2\theta \left(\frac{g_1}{4\pi}\right)^4 \cdots \left(\frac{g_N}{4\pi}\right)^4 \frac{M^4}{16\pi^2}, \qquad m_\phi^2 \sim \sin^2\theta \left(\frac{g_1}{4\pi}\right)^4 \cdots \left(\frac{g_N}{4\pi}\right)^4 M^2 . \tag{2.4}$$

Note that if we send $N \to \infty$ while holding M fixed, supersymmetry is restored. For the non-abelian case we may contemplate sending $M \to \infty$, holding the $SU(k)_i$ scales Λ_i fixed. We can take the $M \to \infty$, $N \to \infty$ limit such that $m_\phi^2 \sim (g^{(1)} \cdots g^{(N)})^4 M^2$ remains fixed. In this case supersymmetry violating effects appear only through the scalar masses m_ϕ, just as in a theory with only soft supersymmetry breaking masses added to the lagrangian. However in this theory an exponentially large hierarchy between these these supersymmetry breaking masses and the cut-off is naturally obtained. For instance, $m_\phi/M \sim 10^{-16}$ for $N \sim 8$ and $g, \theta \sim 1$.

The condition $|h_i|, |h_i'| = g_i$ was assumed for the bare couplings. At energies beneath the cut-off, these couplings run, and will no longer be equal even in magnitude. The differences, like all supersymmetry breaking effects, will be proportional to J, and vanish exponentially for large-N. How then does a low energy observer distinguish supersymmetry breaking via a global twist in theory space from small explicit "local" supersymmetry breaking? The observer would simply run the couplings to higher energies and notice that there is *some* scale at which the magnitude of all $2N$ Yukawa couplings are equal to the corresponding gauge couplings.

3. Relation to Scherk-Schwartz

So far we have discussed the consequences of twisted supersymmetry near the origin of the classical moduli space. It is interesting to explore what happens around a generic point in the classical moduli space, which may include regions where the theory space turns into extra dimensions. In particular, since our supersymmetry breaking is associated with a phase around the circle, we expect that twisting reduces to Scherk-Schwartz supersymmetry breaking in those regions of classical moduli space where extra dimensions are generated. Consider the $\mathrm{U}(1)^N$ model, and expand the theory around the classical vacuum where $\Phi_1 = \cdots = \Phi_N = \phi$. In the supersymmetric limit, this generates a fifth dimension. In the presence of a non-trivial phase θ, the spectrum of the theory already exhibits bose-fermi splitting at tree-level. Since the phase only appears in the gaugino Yukawa couplings, the bosonic spectrum is unaffected. But the fermion mass terms $\lambda M_F \psi$ will have phases. By making phase redefinitions, we can always choose $h_i^*, h_i' = g_i e^{i\theta/N}$. Then the mass squared matrix for the fermions $(M_F^2) \equiv M_F^\dagger M_F$ becomes

$$(M_F^2)_{ij} = \frac{1}{a^2}\left(2\delta_{i,j} - e^{-2i\theta/N}\delta_{i,j+1} - e^{2i\theta/N}\delta_{i,j-1}\right). \tag{3.1}$$

The spectrum of bosons and fermions in the theory is then

$$m_{B,n}^2 = \left(\frac{2}{a}\right)^2 \sin^2\left(\frac{na}{2R}\right), \qquad m_{F,n}^2 = \left(\frac{2}{a}\right)^2 \sin^2\left(\frac{na}{2R} + \theta\right). \tag{3.2}$$

For the modes lighter than $1/a$, this is precisely the spectrum from Scherk-Schwartz supersymmetry breaking in a fifth dimension where the fermions pick up a phase $e^{2i\theta}$ around the circle.

How is this tree-level non-supersymmetric bose-fermi splitting consistent with our previous analysis, which argued that all supersymmetry breaking effects must involve J? To clarify this we examine the bose-fermi mass splitting for the lightest mode in the small θ limit:

$$m_{F,0}^2 - m_{B,0}^2 \simeq \frac{4}{a^2}\theta^2 = 4g^2\phi^2\theta^2. \tag{3.3}$$

This does not appear to vanish like g^{2N} as our previous arguments would seem to suggest. But consider the case of general gauge couplings g_i, not all equal. In this case the mass splitting is

$$m_{F,0}^2 - m_{B,0}^2 \simeq \frac{4N(g_1^2\phi^2)\cdots(g_N^2\phi^2)\theta^2}{\sum_{k=1}^N (g_1^2\phi^2)\cdots\widehat{(g_k^2\phi^2)}\cdots(g_N^2\phi^2)}, \tag{3.4}$$

where the hatted term is to be excluded from the product. Here the numerator is explicitly proportional to $(\mathrm{Re}\, J - g_1^2 \cdots g_N^2)$ just as are the UV sensitive contributions of the previous section, and supersymmetry breaking still vanishes if any one gauge coupling is zero. However in this case there are new *infrared* mass scales, set by $g_i\phi$ which appear in the denominator, and enhance the effect. This phenomenon is well-known and ubiquitous. For example in the Standard Model an invariant measure of CP violation, proportional to many small Yukawa couplings, is of order 10^{-20}. This is enhanced by small infrared mass differences to produce the observed CP violation in the Kaon system of order 10^{-3}.

In the $SU(k)^N$ example, along the directions of classical moduli space where $\Phi_i = \phi_i \mathbf{1}_k$, tree-level bose-fermi mass splittings may also be produced. Here the fermi-bose mass splitting for the lightest mode in the small θ limit is

$$m_{F,0}^2 - m_{B,0}^2 \simeq \frac{4N(g_1^2\phi_1^2)\cdots(g_N^2\phi_N^2)\theta^2}{\sum_{k=1}^N (g_1^2\phi_1^2)\cdots\widehat{(g_k^2\phi_k^2)}\cdots(g_N^2\phi_N^2)}. \tag{3.5}$$

Note that if any one of the ϕ_i vanishes, bose-fermi degeneracy is restored at tree level. This highlights the global nature of the supersymmetry breaking — only in regions of classical moduli space where all links are non-zero does tree-level supersymmetry breaking appear.

Since supersymmetry is broken, we do not expect the classical moduli space to remain flat; quantum corrections will generate a potential for the moduli. For the $\mathrm{U}(1)^N$ example the UV contributions of the previous section generate a quadratic potential $m_\phi^2\phi^2$ with $m_\phi^2 \sim g^{2N}M^2$. On the other hand, away from the origin of classical moduli space there are tree-level bose-fermi splittings proportional to ϕ, and so a Coleman-Weinberg potential for ϕ is generated already at 1-loop. Since all UV sensitive contributions appear first at N loops, the 1-loop Coleman-Weinberg potential cannot contain UV divergences for $N > 2$, with at most log divergences for $N = 2$. This can be verified by computing the 1-loop potential explicitly. The result is indeed cut-off independent for $N > 2$ and is given by [8]:

$$V_{1-\mathrm{loop}}(\phi) = -\frac{3g^4\phi^4}{4\pi^2}\sum_{j=1}^{\infty}\frac{\sin^2(nN\theta)}{n(n^2N^2-1)(n^2N^2-4)}\,. \tag{3.6}$$

This corresponds to the $1/R^4$ potential that is generated for the radius modulus in Scherk-Schwartz models. If m_ϕ^2 is negative, ϕ rolls off to the cut-off M, while if the sign is positive, the origin is meta-stable, with scalar masses hierarchically smaller than M.

The $SU(k)^N$ model is similar. If all the ϕ_i are non-zero a 1-loop Coleman-Weinberg potential is generated. If any one of the ϕ_i are zero, there is no tree-level bose-fermi splitting and the 1-loop potential vanishes. In this case the leading infrared contribution to the potential first appears at 3 loops. In general if r of the ϕ_i are zero this potential first appears at $2r + 1$ loops.

The negative Coleman-Weinberg potential in the $\mathrm{U}(1)^N$ model arose because only the fermionic spectrum was modified by θ. The addition of other degrees of freedom, such as "hopping" chiral superfields, can affect both bosons and fermions. In this case the Coleman-Weinberg potential can be positive. We may also construct models in which the ϕ flat direction is lifted at tree level by additional interactions. These theories may then have stable minima with $g\phi \sim g^N M$, hierarchically below M. This dynamically generates an extra dimension with Scherk-Schwartz supersymmetry breaking, but with a stabilized radius hierarchically larger than M^{-1}.

Scherk-Schwartz supersymmetry breaking via boundary conditions around a fifth dimension, although explicit, is usually said to be very soft at high energies, and therefore insensitive to the physics at the cut-off of the five-dimensional theory [21, 22, 23]. It is most convenient to use position space for the fifth dimension and momentum space for the remaining dimensions. Just as finite temperature boundary conditions introduce an exponential Boltzmann factor, compactification on a circle of radius R with Scherk-Schwartz boundary conditions introduces factors $\exp(-|p_4|R)$, in (Euclideanized) Feynman diagrams involving supersymmetry breaking cutting off loop momenta at $1/R$. This makes supersymmetry breaking radiative corrections finite and calculable, up to effects from higher-dimension local operators power-suppressed by the cut-off. Divergences are associated with the *short distance* structure of the theory and, provided that the physics at the cut-off Λ is local on the scale Λ^{-1}, the breaking of supersymmetry by boundary conditions can not introduce new divergences.

In momentum space for the fifth dimension the essential physics of locality is obscured. Since the higher-dimensional physics becomes strongly coupled, not all the Kaluza-Klein modes can be reliably included in computing radiative corrections, but sharply cutting off the KK sum at some maximum mode gives UV-divergent supersymmetry breaking effects even at low loop order. However, sharp momentum cut-offs also give rise to power-suppressed (rather than exponentially suppressed) non-local effects in position space. If any fully local cut-off procedure is used, these divergent effects disappear.

The situation is especially clear in our concrete UV completions of these models. In order to mimic a continuum field-theoretic dimension, we take the large-N limit. For any finite N, the KK tower is finite, containing only N modes, and there are no UV divergent supersymmetry breaking effects at low loop order. In the KK sum this follows from cancellations associated with the specific $\sin^2(na/2R), \sin^2(na/2R+\theta)$ spectrum. But the momentum space KK calculation is a cumbersome way to deduce what is obvious in theory space (transmuted here into an extra dimension), guaranteed by "local" supersymmetry.

There is a more interesting puzzle here. While there are no UV divergent effects at low-loop order, there *are* power-UV divergent effects at N loops. What is the interpretation of these effects from the low-energy, five-dimensional point of view? In taking the large-N limit we wish to keep the effective five-dimensional gauge coupling $1/g_5^2$ and the compactification radius R fixed. Since the five-dimensional gauge coupling is $1/g_5^2 = 1/(ag^2)$, this keeps the dimensionless ratio $g_5^2/R = g^2/N \equiv g_{\rm LE}^2$ fixed, and the infinite N limit pushes the theory into strong coupling where our picture of the physics breaks down. Nevertheless we can take $g \sim 4\pi$ corresponding to $N\alpha_{\rm LE}/4\pi \sim 1$. The power UV divergent effects for e.g. scalar masses are

$$m_\phi^2 \sim \left(\frac{\alpha}{4\pi}\right)^N M^2 \sim \left(\frac{\alpha_{\rm LE} N}{4\pi}\right)^N M^2\,. \tag{3.7}$$

As N is increased holding $\alpha_{\rm LE}$ fixed, this function decreases till $\alpha_{\rm LE}N \sim 1$, beyond which it increases again; but beyond this point we don't trust our picture of the physics anyway. So, taking N as large as we can to best mimic the continuum extra dimension, the size of the power-supersymmetry violating effects is of order

$$m_\phi^2 \sim e^{-4\pi/\alpha_{\rm LE}} M^2 \sim e^{-2\pi R\Lambda_5} M^2\,, \tag{3.8}$$

where $\Lambda_5 \sim 16\pi^2/g_5^2$ is the naive scale at which the low-energy five-dimensional theory becomes strongly coupled.

In matching to the five-dimensional low energy theory, there is a non-local contribution to the scalar mass, deriving from the non-local breaking of supersymmetry in the UV theory. As befits a non-local effect, it is exponentially small $\sim e^{-R\Lambda_5}$. In the usual effective field theory with extra dimensions, it is assumed that the five dimensional cut-off Λ_5 sets *all* the short-distance scales in the theory, so the coefficient of such a non-local effect would be Λ_5^2 and it's size would be miniscule. There are many UV completions where such an expectation is justified, and in our UV completion the same conclusion is reached if we choose $M \sim \Lambda_5$.

However, in our UV completion, just as the extra dimension itself has no fundamental significance, neither does the naïve five dimensional cut-off. The true UV cut-off M can be much higher than Λ_5. In the limit of exact supersymmetry, the presence of the higher cutoff M would be invisible to the low-energy five dimensional observer. But, in the presence of the explicit supersymmetry breaking associated with the compactification of the extra dimension, the much higher energy scales that were hidden due to supersymmetric cancellations become relevant, and the size of the supersymmetric breaking is much larger than the low-energy five dimensional observer would expect to be associated with a large-distance effect in the fifth dimension.

This violation of low-energy expectations for the size of supersymmetry breaking is intriguing, and provides a toy example of part of a scenario advocated by Banks to address the cosmological constant problem [24]. In Banks' picture all of supersymmetry breaking is cosmological in origin, associated with a deSitter space with a curvature of order the present Hubble radius. The phenomenological problem is then to obtain supersymmetry breaking mass splittings much larger than the naïive classical expectation $\sim 10^{-3}\,\mathrm{eV}$. Radiative corrections seem unable to produce such large splittings, being only logarithmically sensitive to the Planck scale. Banks conjectures that, because of now inexact supersymmetric cancellations, very high energy states above the Planck scale contribute and dramatically modify the size of the scalar masses. As we have seen, exactly this physics occurs in our models, if the "breaking of supersymmetry from deSitter space" is replaced by "breaking of supersymmetry by boundary conditions in a fifth dimension", and if the Planck scale is replaced by the naive five dimensional cutoff Λ_5. A related toy example of this sort was presented in [25].

4. "Local" supersymmetry and Homology

In our previous models, the requirement of "local" supersymmetry was simply satisfied, and the supersymmetry breaking rephase invariant was easy to identify. We now wish to extend our discussion to a general theory characterized by a theory space. As before, we consider assigning arbitrary phases in the gauge-Yukawa couplings involving the link fields.

It will prove convenient to define a *directed* link $l = (ij)_\alpha$ with a line pointing from i to j. The gauge-Yukawa couplings for the components of the chiral superfields Φ_l associated with the link $(i,j)_\alpha$ are

$$\sqrt{2}\phi_l^\dagger h_l \lambda_i \psi_l + \sqrt{2}\phi_l^\dagger h'_l \lambda_j \psi_l \,. \tag{4.1}$$

Once again, we will demand that the magnitudes $|h_l| = g_i$ and $|h'_l| = g_j$, but allow these couplings to have phases. Under field rephasings the product $(h_l^* h'_l)$ is only affected by the gaugino rephasings $\lambda_i \to \omega_i \lambda_i$, transforming as

$$(h_l^* h'_l) \longrightarrow \omega_i^* (h_l^* h'_l) \omega_j \,. \tag{4.2}$$

This motivates the association of a phase U_l with each l, defined as

$$h_l^* h_l' \equiv g_i g_j U_l \,. \tag{4.3}$$

Under the gaugino rephasings, U_l transforms as

$$U_l \longrightarrow \omega_i^* U_l \omega_j \,. \tag{4.4}$$

Note that U_l transforms just as a link variable for a $\mathrm{U}(1)_R$ lattice gauge theory, under which the gauginos on the sites are charged.

Suppose that by gaugino rephasings we could set all $U_l = 1$. Then, h_l, h_l' have the same phase for every link, and we could remove this phase by rephasing the components of Φ_l, without affecting any of the other couplings in the theory. Hence any breaking of supersymmetry must be associated with non-trivial rephase invariants constructed from the U_l. From the lattice gauge theory analogy, we expect these invariants to be associated with closed Wilson lines. We can characterize the rephase invariants precisely by using elementary notions from topology. Let $(l_1, l_2, \ldots, l_{N_l})$ be a list of all the directed links in theory space. The most general candidate for an invariant phase is a product of U_ls of the form $U_{l_1}^{a_1} U_{l_2}^{a_2} \cdots U_{l_{N_l}}^{a_{N_l}}$, where the as are integers. Define a "chain" c to be a formal sum of the form $c = a_1 l_1 + a_2 l_2 + \cdots a_N l_N$, and define a function on chains

$$V(c) = U_{l_1}^{a_1} U_{l_2}^{a_2} \cdots U_{l_{N_l}}^{a_{N_l}} \,. \tag{4.5}$$

It follows that for two chains c_1, c_2, $V(c_1)V(c_2) = V(c_1 + c_2)$. One trivial set of cs, for which $V(c) = 1$, are those traversing a path both forwards and backwards, e.g. $c = (ij)_\alpha + (ji)_\alpha$. We will therefore define $-(ij)_\alpha = (ji)_\alpha$ so that all such chains vanish. In order to characterize the non-trivial rephase invariants, it is helpful to define a linear "boundary" operator ∂ which acts on a directed link field $l = (ij)_\alpha$ as $\partial\,(ij)_\alpha = (j) - (i)$, where $(j) - (i)$ is also understood as a formal sum. It is then easy to see that $V(c)$ is rephase invariant if and only if c is a "closed" chain, satisfying $\partial c = 0$. The associated rephase invariant combination of couplings whose phase is $V(c)$ is

$$J(c) = \tilde{g}_{l_1}^{2a_1} \cdots \tilde{g}_{l_{N_l}}^{2a_{N_l}} V(c) \,, \tag{4.6}$$

where for each $l = (ij)_\alpha$ we define $\tilde{g}_l^2 \equiv g_i g_j$.

Exact supersymmetry requires $V(c) = 1$ for all closed chains c. But we wish to impose the weaker condition of "local" supersymmetry. To do this, we need to make a choice for some set of sites and links that are "locally connected". Such a choice can be specified by a non-zero closed chain of the form $(i_1 i_2)_{\alpha_1} + (i_2 i_3)_{\alpha_2} + \cdots + (i_{N_p} i_1)_{\alpha_{N_p}}$ which includes the "locally connected" sites and links. We define a plaquette p to be an object $p = (i_1 i_2)_{\alpha_1} (i_2 i_3)_{\alpha_2} (i_{N_p} i_1)_{\alpha_{N_p}}$, whose boundary is $\partial p = (i_1 i_2)_{\alpha_1} + (i_2 i_3)_{\alpha_2} + \cdots + (i_{N_p} i_1)_{\alpha_{N_p}}$. Just as we defined a chain c to be a formal sum of directed links, we define a surface s to be a formal sum of plaquettes.

We can now state the condition of "local" supersymmetry precisely: we require that the phases from any given "locally connected" set of sites and links characterized by p can be removed by rephasing the fields. If $V(\partial p) \neq 1$, this is impossible. So we require

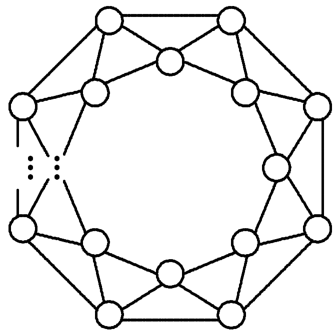

Figure 4: Ribbon theory space

that for all plaquettes p, $V(\partial p) = 1$. Since $V(c_1 + c_2) = V(c_1)V(c_2)$, this immediately implies that for any surface s, $V(\partial s) = 1$. Therefore, if we have two closed chains c_1, c_2 such that $c_1 - c_2 = \partial s$ for some surface s, $V(c_1) = V(c_2)$. This is also familiar from the lattice analogy. The requirement of "local" supersymmetry in this case is the absence of $\mathrm{U}(1)_R$ flux through the plaquettes, and therefore if two closed paths form the boundary of a region consisting of plaquettes, the associated Wilson loops will be equal.

This motivates the introduction of an equivalence relation between closed chains where c_1, c_2 are "homologous" $c_1 \sim c_2$ if $c_1 - c_2 = \partial s$ for some s. Then for every equivalence class $[c]$ of homologous chains there is an associated phase $V([c])$ common to all the chains in $[c]$. These equivalence classes form an additive group with the addition rule $[c_1] + [c_2] = [c_1 + c_2]$, the first Homology group $H_1(T, P)$ associated with the theory space T and choice of plaquettes P, and the phases $V([c])$ must be a representation of $H_1(T, P)$. If $H_1(T, P)$ is trivial, then the requirement of "local" supersymmetry is enough to ensure exact supersymmetry; if not we can introduce supersymmetry breaking in a way consistent with "local" supersymmetry.

Let us look at a few examples, beginning with our simple circular theory spaces. All closed chains are of the form nc_1 where $c_1 = (12) + (23) + \cdots (N1)$. The only possible plaquette is $p = (12)(23) \cdots (N1)$ with boundary $\partial p = c_1$. The possible choices for P are then either the empty set or p. Clearly the latter does not correspond to any reasonable notion of locality, since with this definition all sites are locally connected even as $N \to \infty$. We therefore choose P to be empty, which is another way of saying that our requirement of "local" supersymmetry is automatic in this theory. Since all closed chains are multiples of c_1 and there are no plaquettes, $H_1(T, P) = Z$ with elements $[nc_1]$, and the phases $V([nc_1]) = e^{in2\theta}$.

Now consider the "ribbon" theory space shown in figure 4. There are many possible choices for the plaquettes in P; the most natural one is the triangles connecting nearest neighbor sites with clockwise orientation, by which we mean $p = l_1 l_2 l_3$ where the links l_1, l_2, l_3 wind clockwise around a triangle in the figure. With this choice it is clear that all closed chains are homologous to multiples of e.g. the chain going around the inner polygon, and again $H_1(T, P) = Z$.

Next, consider the theory space of figure 5; the presence of the group in the center makes the theory "non-local" in theory space, in the sense that every point is connected to every other point through at most two links. If we choose the plaquettes to contain the

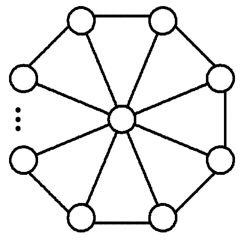

Figure 5: A non-local theory space.

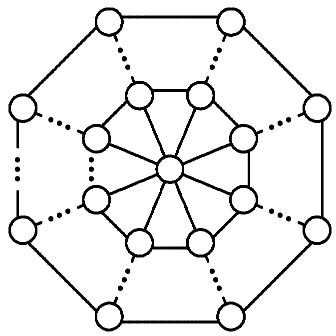

Figure 6: Spider web theory space.

triangles as before, the theory space is topologically a disk and all chains are homologous to zero, so no supersymmetry breaking phases are allowed. If we chose P to be empty instead, in addition to the non-trivial closed chains winding around the circle, there are new "small" chains winding around the individual triangles, and the supersymmetry breaking rephase invariants are unsuppressed as we take $N \to \infty$. Note that the rephase invariants associated with the small cycles are all proportional to the gauge coupling of the center group, so that these supersymmetry breaking invariants vanish as this gauge coupling vanishes. In this limit the center point is effectively removed from the theory space and we revert to the circular model.

Finally, consider the theory space of figure 6, consisting of a site at the center with $2N$ lines radiating outward, intersecting k concentric circles each with $2N$ sites.

The most natural choice of plaquettes consists of all clockwise oriented triangles and squares in the figure. As it stands, this space is also topologically a disk with a trivial $H_1(T, P)$. However, we can make the model more interesting by identifying diametrically opposite sites (and the links connecting them) on the boundary, i.e. identifying the site i with $i+N$ on the kth circle. The resulting space is topologically the same as the projective plane RP^2, with a first homology group $H_1(RP^2) = Z_2$. We can see this directly as follows. All closed chains are clearly homologous to multiples of the chain c_1 that traverses the circumference of the largest circle, starting from a given site i to $i+N$. Consider the boundary ∂s of the surface s which is the sum of all the plaquettes in the theory. The contribution to ∂s from all the links on the interior of the diagram cancel, and we are left with the sum of the links on the outside, which is equal to $2c_1$. That is, $2c_1 = \partial s$, and so $[2c_1] = [0]$. Therefore $H_1(T, P) = Z_2$, and the only possible supersymmetry breaking rephase invariant is

$V([c_1]) = -1$. This is pleasing: in our examples with a circle, the supersymmetry violating phase was arbitrary. Here, the requirement of "local" supersymmetry either demands exact supersymmetry or a maximal supersymmetry violating phase. This construction clearly generalizes: if we take mN sites on the boundary circle and again identify the sites i with $i+N$, the homology group is Z_m, and the rephase invariant phases are the mth roots of unity.

5. Twisting $\mathcal{N} = 2$ supersymmetry

Supersymmetry may be twisted in $\mathcal{N} = 2$ models as well. Including phases in the gauge-Yukawa couplings as before, the $\mathcal{N} = 2$ supersymmetry would break all the way down to $\mathcal{N} = 0$. But we can also construct models which leave an $\mathcal{N} = 1$ supersymmetry intact. Consider the model of figure 1 as an $\mathcal{N} = 2$ supersymmetric $\mathrm{U}(1)^N$ field theory. In this case each link corresponds to a hypermultiplet which, in $\mathcal{N} = 1$ language, contains two chiral multiplets H^c, H, while the vector multiplet at each site contains a chiral adjoint superfield X. The superpotential is

$$W = \sum_{i=1}^{N} \sqrt{2} H_i^c \left(f_i X_i - f'_{i-1} X_{i-1} \right) H_i \tag{5.1}$$

$\mathcal{N} = 2$ supersymmetry requires that these Yukawa couplings are equal to the gauge couplings: $f_i, f'_i = g_i$. Just as in our $\mathcal{N} = 1$ example, we allow for arbitrary phases in these couplings. By the same arguments used in the $\mathcal{N} = 1$ example, we see that the combination of couplings analogous to J is the unique rephase invariant measure of supersymmetry breaking in these couplings, in this case from $\mathcal{N} = 2$ to $\mathcal{N} = 1$. The exact $\mathcal{N} = 1$ supersymmetry eliminates any power sensitivity to the UV, but the $\mathcal{N} = 2$ non-renormalization theorems will be violated at N-loop order. For example $\mathcal{N} = 2$ supersymmetry forces the hypermultiplet wave-function renormalization to vanish. Here, the wave-function renormalization is non-zero, but small:

$$\log Z_H \sim \left(\frac{\alpha}{4\pi} \right)^N . \tag{5.2}$$

Note that the generalization to $\mathcal{N} = 2$ $SU(k)^N$ theory is a conformal field theory before twisting. Upon twisting the conformal symmetry is broken, but only through wave-function renormalization which appears at N loops! This provides a natural example of a "walking" gauge theory, with gauge couplings evolving arbitrarily slowly beneath the cut-off.

We can also break $\mathcal{N} = 2$ to $\mathcal{N} = 0$ in a different way. Imagine removing the chiral multiplet X_i from the ith site on the chain. This would break the supersymmetry down to $\mathcal{N} = 1$. Alternatively we could remove the complex scalar component of X_i and the gaugino component λ_i of the vector multiplet on this site. This still preserves an $\mathcal{N} = 1$ supersymmetry, where the fermionic component of X_i takes the place of the gaugino. But now consider a finite chain of N gauge groups rather than a circle. If we remove the chiral multiplet X_N from the rightmost site *as well as* the complex scalar from X_1 and the gaugino λ_1 from the leftmost site we completely break supersymmetry. Locally we preserve an $\mathcal{N} = 2$ supersymmetry, where the two end sites each preserve a different $\mathcal{N} = 1$ subgroup of the $\mathcal{N} = 2$ supersymmetry. Consequently fully non-supersymmetric effects must involve all couplings from each site, again appearing only at N loops.

6. Conclusions and outlook

The exploration of theory space has revealed surprising new phenomena. In special cases, field-theoretic extra dimensions with their familiar dynamics arise from theory space at low energies. More generally there are new features that are intrinsic to theory space itself. Supersymmetric field theories provide a particularly clear illustration of this point. Only in some regions of classical moduli space does theory space transmute into extra dimensions, but the underlying theory space plays a deeper role. The structure of sites and links together with a notion of "locality" endows theory space with a topological significance irrespective of the choice of vacuum. If this topology is non-trivial it is possible to preserve supersymmetry locally in theory space, but break it through topological obstructions. This breaking manifests itself differently in different regions of the classical moduli space. At the origin supersymmetry breaking effects appear only at N-loop order, while in regions of classical moduli space where the entire theory space reduces to extra dimensions the breaking appears at tree-level, reducing to the Scherk-Schwartz mechanism of supersymmetry breaking. In general regions of classical moduli space only certain areas of the theory space reduce to extra dimensions, and supersymmetry breaking effects involve perturbative loop factors for all links outside these areas, interpolating between the unsuppressed breaking of extra dimensions and the exponentially small breaking of bare theory space.

Even when twisted supersymmetry reduces to Scherk-Schwartz breaking, theory space gives new insight. The models presented here provide a UV completion [5] of the Scherk-Schwartz mechanism, allowing investigation of the physics at energies near and above the naïve cut-off of the higher-dimensional theory.

Twisted supersymmetry breaking has obvious phenomenological applications. When theory space reduces to extra dimensions, twisted supersymmetry reduces to standard Scherk-Schwartz supersymmetry breaking with the potential added benefit of naturally generating a large hierarchy between the compactification scale $1/R$ and the cut-off M. But non-extra-dimensional examples are also possible. We may imagine a theory space in which the non-trivial twist resides in some portion while the Standard Model fields reside in another. Supersymmetry breaking then appears to be communicated to the Standard Model from a "hidden sector". Since twisted supersymmetry breaking effects are suppressed by many loop factors, this can also generate a large hierarchy.

We have so far ignored gravity. If we naïvely add four-dimensional supergravity, these interactions act non-locally in theory space and therefore supersymmetry breaking need not involve the large number of gauge couplings present in our rephase invariants. For example there are direct gravitational couplings of twisted sector supersymmetry breaking to Standard Model fields which gives Standard Model scalar masses of size

$$m^2 \sim \frac{M^6}{M_{\text{Planck}}^4}\left(\frac{\alpha}{4\pi}\right)^2 . \tag{6.1}$$

For the UV scale M smaller than $M_{\rm Planck}$ these effects can be quite small, even for M far above the TeV scale. For example for $\alpha/(4\pi) \sim 10^{-2}$, these masses are smaller than 1 TeV for all $M < 10^{11}$ TeV. Gravitational interactions will also generate supersymmetry breaking scalar masses directly in the twisted sector, of size

$$m_\phi^2 \sim \frac{M^4}{M_{\rm Planck}^2}\left(\frac{\alpha}{4\pi}\right)^2. \tag{6.2}$$

These in turn will generate supersymmetry breaking in the Standard Model sector through the links in theory space that connect the Standard Model to the twisted sector. Again, even for a small number of links and a moderate N these effects are smaller than the direct effects from twisted supersymmetry.

In previous work we used theory space to accelerate the supersymmetric unification of gauge couplings to much lower energies. It is therefore natural to try and combine accelerated unification with twisted supersymmetry. There is also a phenomenological hint to do so. On the one hand, gauge coupling unification suggests a high fundamental scale. On the other, if supersymmetry breaking is present at such a high scale, radiative corrections to the Higgs mass from top squarks are enhanced by a large logarithm, so that natural electroweak symmetry breaking suggests that superpartners should already have been observed. The combination of accelerated unification and twisted supersymmetry breaking can alleviate the tension between these two facts, since the couplings unify at a much lower scale, and supersymmetry breaking can arise even at tree-level in the case where an extra dimension forms. In this case, accelerated unification arises from scales above those where an extra dimension forms, while supersymmetry breaking arises from beneath this scale. The ability to address high energy scales is thus crucial.

Our excursions in theory space [5, 8, 9] have thus far been motivated by specific physical problems. Twisted supersymmetry breaking is yet another illustration of the unexpected physics that can arise from the interplay of sites and links. The surprising ease with which theory space has provided new approaches to problems as diverse as UV completion of higher-dimensional gauge theories, stabilization of the electroweak scale, low scale gauge coupling unification, and supersymmetry breaking, suggests to us that there are deeper ideas surrounding these structures that we have yet to fathom.

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